J Stat Phys (2009) 136: 131–143
DOI 10.1007/s10955-009-9773-6

Stationary Properties and Stochastic Resonance for a Saturation Laser Model with Cross-correlation Between Quantum Noise Terms

Ping Zhu · Yang Fu

Received: 4 October 2008 / Accepted: 15 June 2009 / Published online: 27 June 2009

Abstract The stationary properties of a saturation laser model with cross-correlation between the real and imaginary parts of the quantum noise are investigated theoretically. Using the Novikov theorem and the Sargent technique, we obtain the analytic expressions of the stationary probability density distribution, the mean, the variance and the skewness of the saturation laser model. The cross-correlation coefficient λ and other parameters can make the stationary probability density distribution $P_{st}(I)$ generate interesting two-extrema structure, one-extremum structure, or no-extremum structure. It is clearly found that a first-order-like-transition is induced by the coupling strength $|\lambda|$ of the complex quantum noise terms in the saturation laser model. When the laser system is operated above the threshold, the mean $\langle I \rangle$ becomes larger and the output of the laser intensity increases; however the coupling strength $|\lambda|$ attenuates the output of the laser intensity. When the laser is operated near and below the threshold, the mean $\langle I \rangle$ becomes smaller, the output of the laser intensity decreases, and $|\lambda|$ still attenuates the output of the laser intensity. When a periodic signal is added to a saturation laser model with cross-correlation between quantum noise terms, the interesting stochastic resonance phenomena occur at $\lambda = 0$. The noise intensity Q decreases the values of the resonance peak, however, the amplitude of the periodic signal B enhances the values of the resonance peak.

Keywords Saturation laser model · Quantum noise terms · Real and imaginary parts · Cross-correlation · Stationary properties · Stochastic resonance

P. Zhu (✉)
Department of Physics, Yunnan University, Kunming 650091, China
e-mail: zhuupp@163.com

P. Zhu
Department of Physics, Simao Teacher's College, Simao 665000, China

Y. Fu
College of Physics and Electronics, Yannan Normal University, Kunming 650092, China

1 Introduction

Statistical fluctuations of laser radiation determine the limits on the use of lasers in almost every application. For a laser system, the effects of the internal quantum noise are important and interesting. Generally speaking, the quantum noise is the complex noise, of which the real and imaginary parts may be non-correlation or cross-correlation. When the real and imaginary parts of the noise terms in the laser field originate from independent noise sources, the real and imaginary parts are non-correlation [1–4]. However, in certain situations, when the fluctuation of the complex field can be coherent and the real and imaginary parts of the quantum noise originate from a common bath, two parts may be cross-correlation [5–8]. A few studies exist in which the effects of cross-correlation between additive and multiplicative noises are considered [9–14]. The statistic properties of a cubic model of single-mode laser with non-correlation or cross-correlation between the real and imaginary parts of the quantum noise have been widely investigated [1–8]. Zhou et al. have taken into account the cross-correlation between real and imaginary parts of the quantum noise in a two-dimensional single-mode laser, specifically in their calculation that the amplitude equation was not decoupled from the phase equation [7]. Employing the technique developed by Sargent III et al., Ke et al. obtained the mean, the variance and the skewness of the laser intensity of a cubic model of single-mode laser with cross-correlation between the real and imaginary parts of the quantum noise [8]. In 2004, Xie and Mei discussed the relaxation time and correlation function of a cubic model of single-mode laser with cross-correlation between the real and imaginary parts of the quantum noise [15] and Zhang et al. yet investigated the case with multiplicative colored noise with cross-correlation between the quantum noise terms [16]. The saturation laser model is as typical as the cubic model of single-mode laser, which possesses important theoretical value and application value [17–22]. First-order-phase transitions are interesting physical phenomena which were already observed in early experiments [23–27]. Lett et al. discussed colored-noise-induced first-order phase transition in a single-mode dye laser [28, 29]. Thereafter, a first-order-like transition induced by the cross-correlation strength $|\lambda|$ of the complex quantum noise in the single-mode laser is shown in [8].

In this paper, we investigate the stationary properties and the stochastic resonance for a saturation laser model with cross-correlation between the real and imaginary parts of the quantum noise from the point of view of phase lock and show the effects of cross-correlation between the real and imaginary parts. In Sect. 2, applying the technique developed by Sargent III et al., the analytic expression of the stationary probability density distribution (SPDD), the mean, the variance, and the skewness of laser intensity are derived. In Sect. 3, adding a periodic signal to a saturation laser model with cross-correlation between quantum noise terms, we discuss the stochastic resonance phenomena. In Sect. 4 the first-order-like transition which is induced by the cross-correlation strength $|\lambda|$ of the complex quantum noise in the saturation laser model is revealed and the effects of λ for the SPDD, the mean, the variance, the skewness of the laser intensity, and the signal-to-noise ratio (SNR) distributions are drawn. Discussions and conclusions conclude the paper.

2 Langevin Equation and Steady-distribution Function

The laser model with a full account of the saturation effects follows the Langevin equation [30]

$$\frac{dE}{dt} = -KE + \frac{F_1 E}{1 + A|E|^2/F_1} + q(t), \tag{1}$$

where the electric field E is complex and dimensionless, K is the cavity decay rate for the electric field, $F_1 = a_0 + K$ is the gain parameter, and a_0 and A are real and stand for the net gain and the self-saturation coefficient, in which the random variable $q(t)$ represents complex quantum noise. The noise terms are assumed to be Gaussian white noise with zero mean and correlations:

$$\langle q_I(t)q_I(t')\rangle = Q\delta(t-t'), \tag{2}$$

$$\langle q_R(t)q_R(t')\rangle = Q\delta(t-t'), \tag{3}$$

$$\langle q_I(t)q_R(t')\rangle = \langle q_R(t)q_I(t')\rangle = \lambda Q\delta(t-t'), \tag{4}$$

where q_I and q_R stand for the real and imaginary parts. The parameter λ, the cross-correlation coefficient, measures the degree of cross-correlation between $q_I(t)$ and $q_R(t)$, and $-1 \le \lambda \le 1$. The absolute value of the cross-correlation coefficient $|\lambda|$ reflects the coupling strength between $q_I(t)$ and $q_R(t)$.

Performing the polar coordinate transform $E = re^{i\phi}$ in (1), where r and ϕ are the amplitude and the phase of the laser field, respectively, we can obtain two-dimensional stochastic equations of the field amplitude and the field phase as follows [8]

$$\frac{dr}{dt} = -Kr + \frac{F_1 r}{1+\frac{Ar^2}{F_1}} + \epsilon_r(t), \tag{5}$$

$$\frac{d\phi}{dt} = \frac{1}{r}\epsilon_\phi(t), \tag{6}$$

where

$$\epsilon_r(t) = q_R(t)\cos\phi + q_I(t)\sin\phi, \tag{7}$$

$$\epsilon_\phi(t) = -q_R(t)\sin\phi + q_I(t)\cos\phi. \tag{8}$$

Since both $\langle\epsilon_r(t)\rangle$ and $\langle\epsilon_\phi(t)\rangle$ are nonzero, applying the techniques developed in [2, 31], $\epsilon_r(t)$ and $\epsilon_\phi(t)$ can be changed to

$$\epsilon_r(t) = \frac{Q}{2r}(1-\lambda\sin 2\phi) + \overline{\epsilon}_r(t), \tag{9}$$

$$\epsilon_\phi(t) = -\frac{Q}{2r}\lambda\cos 2\phi + \overline{\epsilon}_\phi(t), \tag{10}$$

where $\overline{\epsilon}_r(t)$ and $\overline{\epsilon}_\phi(t)$ are pure noises with zero mean and correlations

$$\langle\overline{\epsilon}_r(t)\overline{\epsilon}_r(t')\rangle = Q(1+\lambda\sin 2\phi)\delta(t-t'), \tag{11}$$

$$\langle\overline{\epsilon}_\phi(t)\overline{\epsilon}_\phi(t')\rangle = Q(1-\lambda\sin 2\phi)\delta(t-t'), \tag{12}$$

$$\langle\overline{\epsilon}_r(t)\overline{\epsilon}_\phi(t')\rangle = \langle\overline{\epsilon}_\phi(t)\overline{\epsilon}_r(t')\rangle = Q\lambda\cos 2\phi\delta(t-t'). \tag{13}$$

Substituting (10) into (6), we have

$$\frac{d\phi}{dt} = -\frac{Q}{2r^2}\lambda\cos 2\phi + \frac{1}{r}\overline{\epsilon}_\phi(t). \tag{14}$$

In (14), a new drift term, $-(Q/2r^2)\lambda\cos 2\phi$, which arises from the nonzero cross-correlation coefficient λ, can lead the laser phase to be locked at a stable value ϕ_s that is determined by [31]

$$\lambda \sin 2\phi_s = |\lambda|, \tag{15}$$

where $\phi_s = (2n+1)\pi/4$, $n = 0, \pm 1, \pm 2, \dots$.

Inserting (15) and (9) into (5), the decoupled amplitude equation can be written as

$$\frac{dr}{dt} = -Kr + \frac{F_1 r}{1+\frac{Ar^2}{F_1}} + \frac{Q}{2r}(1-|\lambda|) + \overline{\epsilon}_r(t), \tag{16}$$

where

$$\langle \overline{\epsilon}_r(t)\rangle = 0, \tag{17}$$

$$\langle \overline{\epsilon}_r(t)\overline{\epsilon}_r(t')\rangle = Q(1+|\lambda|)\delta(t-t'). \tag{18}$$

Defining the laser intensity as I, we have $I = |E|^2 = r^2$ and finally obtain the Langevin equation on I corresponding to (16)

$$\frac{dI}{dt} = -2KI + \frac{2F_1 I}{1+\frac{AI}{F_1}} + Q(1-|\lambda|) + 2I^{\frac{1}{2}}\overline{\epsilon}_r(t). \tag{19}$$

The Fokker-Planck equation (FPE) for the probability density distribution $P(I,t)$ of the laser intensity corresponding (19) is given by [5, 32]

$$\frac{\partial P(I,t)}{\partial t} = -\frac{\partial}{\partial I}[F(I)P(I,t)] + \frac{\partial^2}{\partial I^2}[B(I)P(I,t)], \tag{20}$$

where

$$F(I) = -2KI + \frac{2F_1 I}{1+AI/F_1} + 2Q, \tag{21}$$

$$B(I) = 2Q(1+|\lambda|)I. \tag{22}$$

In the case of a stationary state, the stationary probability density distribution (SPDD) $P_{st}(I)$ can be obtained directly from (20) and is given by

$$P'_{st}(I) = \frac{N'}{2Q(1+|\lambda|)} I^{\beta_1}\left(1+\frac{A}{F_1}I\right)^{\beta_2} \exp(\beta_3 I) \lim_{\varepsilon\to 0}\frac{1}{\varepsilon^{\frac{1}{1+|\lambda|}}} \tag{23}$$

for $-1 \le \lambda \le 1$, where ε is a positive infinitesimal quantity of the laser intensity I near the zero,

$$\beta_1 = -\frac{|\lambda|}{1+|\lambda|},$$

$$\beta_2 = \frac{F_1^2}{AQ(1+|\lambda|)},$$

$$\beta_3 = \frac{-K}{Q(1+|\lambda|)},$$

and N' is a integral constant.

Then from (23), we get the normalized stationary probability density function

$$P_{st}(I) = \frac{N}{2Q(1+|\lambda|)} I^{\beta_1}\left(1+\frac{A}{F_1}I\right)^{\beta_2} \exp(\beta_3 I), \quad (24)$$

where N is the normalization constant which is given by

$$N = \frac{1}{\int_0^\infty \frac{1}{2Q(1+|\lambda|)} I^{\beta_1}(1+\frac{A}{F_1}I)^{\beta_2} \exp(\beta_3 I) dI}.$$

Then expectation values of the nth power of the laser intensity I are given by

$$\langle I^n \rangle = \int_0^\infty I^n P_{st}(I) dI. \quad (25)$$

The mean, the normalized variance, and the skewness of the laser intensity can be given by the following.

The mean laser intensity is

$$\langle I \rangle = \int_0^\infty I P_{st}(I) dI. \quad (26)$$

The normalized variance of intensity is

$$\lambda_2(0) = \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1. \quad (27)$$

And the normalized skewness is

$$\lambda_3(0) = \frac{\langle I^3 \rangle}{\langle I \rangle^3} - 3\lambda_2(0) - 1. \quad (28)$$

3 Stochastic Resonance

The equation of the intensity for the saturation laser model with cross-correlation between quantum noise terms added a input signal is given by

$$\frac{dI}{dt} = -2KI + \frac{2F_1 I}{1+\frac{AI}{F_1}} + Q(1-|\lambda|) + 2I^{\frac{1}{2}}\overline{\epsilon}_r(t) + B\cos(\Omega t), \quad (29)$$

where B and Ω are the amplitude and the frequency of the harmonic excitation, respectively.

Linearizing (29) around the deterministic steady-state intensity $I_0 = F_1(F_1 - K)/KA$ and writing $I = I_0 + \delta$ we have [33]

$$\frac{d\delta}{dt} = -\gamma\delta + Q(1-|\lambda|) + 2I^{\frac{1}{2}}\overline{\epsilon}_r(t) + B\cos(\Omega t), \quad (30)$$

where $\gamma = 2K(F_1 - K)/F_1$.

The normalized correlation function defined [34] by

$$C(t) = \lim_{t' \to \infty} \frac{\langle I(t'+t)I(t')\rangle - \langle I(t')\rangle^2}{\langle I(t')\rangle^2} \tag{31}$$

is obtained by straightforward integration of (29) and making use of (17) and (18). And than we find

$$C(t) = 2\frac{Q(1-|\lambda|)}{\gamma I_0} + \left(\frac{Q(1-|\lambda|)}{\gamma I_0}\right)^2 + \frac{2Q(1+|\lambda|)\exp(-\gamma t)}{\gamma I_0} + \frac{B^2}{2I_0^2(\gamma^2+\Omega^2)}\cos(\Omega t). \tag{32}$$

The power spectrum $S(\omega)$ of the intensity fluctuations defined as the Fourier transform of $C(t)$ becomes [35]

$$\begin{aligned} S(\omega) &= \left[2\frac{Q(1-|\lambda|)}{\gamma I_0} + \left(\frac{Q(1-|\lambda|)}{\gamma I_0}\right)^2\right] 2\pi\delta(\omega) + \frac{4Q(1+|\lambda|)}{I_0(\gamma^2+\omega^2)} \\ &\quad + \frac{B^2\pi}{2I_0^2(\gamma^2+\Omega^2)}\left[\delta(\omega-\Omega)+\delta(\omega+\Omega)\right] = S_1(\omega) + S_2(\omega), \end{aligned} \tag{33}$$

where

$$S_1(\omega) = \left[2\frac{Q(1-|\lambda|)}{\gamma I_0} + \left(\frac{Q(1-|\lambda|)}{\gamma I_0}\right)^2\right] 2\pi\delta(\omega) + \frac{4Q(1+|\lambda|)}{I_0(\gamma^2+\omega^2)},$$

and

$$S_2(\omega) = \frac{B^2\pi}{2I_0^2(\gamma^2+\Omega^2)}\left[\delta(\omega-\Omega)+\delta(\omega+\Omega)\right].$$

$S_1(\omega)$ and $S_2(\omega)$ are the output power spectrum of noise and signal, respectively.

The signal-to-noise ratio (SNR) is defined as

$$R = \frac{P_s}{S_1(\omega=\Omega)}, \tag{34}$$

where $P_s = \int_0^\infty S_2(\omega)d\omega$. We have

$$R = \frac{KB^2A\pi}{8(K+a_0)a_0Q(1+|\lambda|)}. \tag{35}$$

4 Discussions and Conclusions

To see the effects of the cross-correlation between the real and imaginary parts of the quantum noise on a saturation laser model under a stable locked phase ϕ_s, we first solve the extrema of the steady-state probability density distribution $P_{st}(I)$. In (24), let $\partial P_{st}(I)/\partial I = 0$, and we have

$$\frac{AK}{F_1}I^2 - \left(F_1 - K - \frac{|\lambda|AQ}{F_1}\right)I + Q|\lambda| = 0. \tag{36}$$

When

$$\left(F_1 - K - \frac{|\lambda| A Q}{F_1}\right)^2 - \frac{4KAQ|\lambda|}{F_1} > 0, \tag{37}$$

the steady-state probability density distribution $P_{st}(I)$ possesses two extrema, which possesses one maximum $P_{stmax}(I)$ and one minimum $P_{stmin}(I)$. When

$$\left(F_1 - K - \frac{|\lambda| A Q}{F_1}\right)^2 - \frac{4KAQ|\lambda|}{F_1} = 0, \tag{38}$$

the steady-state probability density distribution $P_{st}(I)$ only possesses one maximum $P_{stmax}(I)$. When

$$\left(F_1 - K - \frac{|\lambda| A Q}{F_1}\right)^2 - \frac{4KAQ|\lambda|}{F_1} < 0, \tag{39}$$

the steady-state probability density distribution $P_{st}(I)$ possesses no-extremum. When the discriminant $\Delta = (F_1 - K - \frac{|\lambda|AQ}{F_1})^2 - \frac{4KAQ|\lambda|}{F_1} > 0$, the laser intensity I_{max} and I_{min} corresponding to the maximum and the minimum of the steady-state probability density distribution $P_{st}(I)$ are respectively given by

$$I_{max,mini} = \frac{(a_0 - \frac{|\lambda|AQ}{a_0+K}) \pm \sqrt{(a_0 - \frac{|\lambda|AQ}{a_0+K})^2 - \frac{4KAQ|\lambda|}{a_0+K}}}{\frac{2AK}{a_0+K}}. \tag{40}$$

Since $|a_0 - \frac{|\lambda|AQ}{a_0+K}| > \sqrt{(a_0 - \frac{|\lambda|AQ}{F_1})^2 - \frac{4KAQ|\lambda|}{F_1}}$, we attain $I_{max,mini} < 0$ for $a_0 \le 0$ which can't accord with the practical situation of $I \ge 0$. Thus when the saturation laser is operated at the threshold and below the threshold, the steady-state probability density distribution $P_{st}(I)$ can't possesses extremal structures for $I > 0$.

Using (24)–(28), we can obtain and discuss the stationary probability density distribution, the mean, the normalized variance, and the normalized skewness of the laser intensity by the numerical calculation.

The discriminant $\Delta = \frac{A^2Q^2}{(a_0+K)^2}|\lambda|^2 - \frac{2AQ}{(a_0+K)}(a_0 + 2K)|\lambda| + a_0^2$ as a function of the cross-correlation coefficient λ is plotted in Fig. 1(a) and the stationary probability distribution $P_{st}(I)$ versus the laser intensity variable for different discriminants which is determined by cross-correlation coefficient λ is plotted in Fig. 1(b). From Fig. 1, we see that when other parameters fixed and $\lambda = \pm 0.1$ make the discriminant $\Delta > 0$, the stationary probability distribution $P_{st}(I)$ possesses one-maximum and one-minimum structure; when other parameters fixed and $\lambda = \pm 0.52$ make the discriminant $\Delta = 0$, the stationary probability distribution $P_{st}(I)$ only possesses one-minimum structure; when other parameters fixed and $\lambda = \pm 0.95$ make the discriminant $\Delta < 0$, the stationary probability distribution $P_{st}(I)$ possesses no-extremum structure. When the laser is operated above the threshold, the SPDD undergoes three different shapes from two-extremum structure to one-extremum structure, again to no-extremum structure as $|\lambda|$ increases from 0 to 1, in which the maximum of the peak of $P_{st}(I)$ decreases, the minimum of $P_{st}(I)$ increases, and finally becomes no-extremum structure. This is a interesting first-order-like-transition phenomenon, the most important of all, which is induced by the coupling strength $|\lambda|$ between the real and imaginary parts of the quantum noise in the saturation laser model under phase lock.

Figure 2 shows that when the laser is operated above the threshold and the parameters satisfy $\Delta > 0$, $P_{st}(I)$ possesses the maximum structure, the height of the peak of $P_{st}(I)$

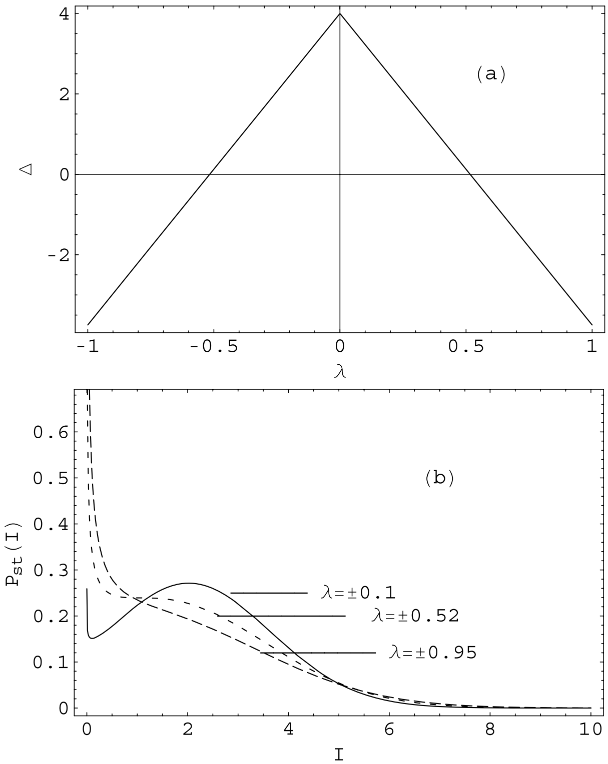


Fig. 1 (**a**) The discriminant Δ as a function of the cross-correlation coefficient λ for $a_0 = 2$, $A = 1$, $Q = 2$ and $K = 30$. (**b**) $P_{st}(I)$ vs. the variable I for three kinds of different discriminants induced by the coupling strength $|\lambda|$. Parameters chosen are $a_0 = 2$, $A = 1$, $Q = 2$ and $K = 30$

decreases as $|\lambda|$ increases. When the laser is operated near the threshold and below the threshold, $P_{st}(I)$ exhibits no-extremum structure; $P_{st}(I)$ always gives a higher value as $I \to 0$, and $P_{st}(I)$ monotonously decreases as I increases and finally has a longer tail before decreasing to zero. The larger $|\lambda|$ is, the more obvious the phenomenon is. When $\lambda = 0$, the $P_{st}(I) - I$ curves are agreeable with the results given by [4].

The mean of the laser intensity $\langle I \rangle$ as a function of the pump parameter a_0 is plotted in Fig. 3. It is obvious that the mean of the laser intensity $\langle I \rangle$ increases as a_0 increases. For near and below the threshold, the mean $\langle I \rangle$ displays almost no difference as λ changes. However, for above threshold, the mean of the laser intensity increases rapidly as a_0 increases and the mean of the laser intensity decreases slightly as $|\lambda|$ increases.

In Fig. 4, the effects of the coupling strength $|\lambda|$ between the real and imaginary parts for the mean of the laser intensity $\langle I \rangle$ is shown. above the threshold, the mean of the laser intensity is the largest, at the threshold secondary, and below the threshold the smallest. The mean $\langle I \rangle$ monotonously decreases as $|\lambda|$ increases.

The normalized variance $\lambda_2(0)$ and the normalized skewness $\lambda_3(0)$ of the laser intensity are plotted against the pump parameter a_0 in Figs. 5 for different values of $|\lambda|$. When the laser is operated well above the threshold, $\lambda_2(0)$ and $\lambda_3(0)$ is almost no difference as $|\lambda|$ changes. When the laser is operated at the threshold and below the threshold, $\lambda_2(0)$ and $\lambda_3(0)$ occur large deviations as $|\lambda|$ changes; $\lambda_2(0) > 0$ and $\lambda_3(0) > 0$ for different pump parameters a_0; the larger the coupling strength $|\lambda|$ is, the larger the values of $\lambda_2(0)$ and $\lambda_3(0)$ are.

When a periodic signal is added to a saturation laser model with cross-correlation between quantum noise terms, the interesting stochastic resonance phenomena occur. The

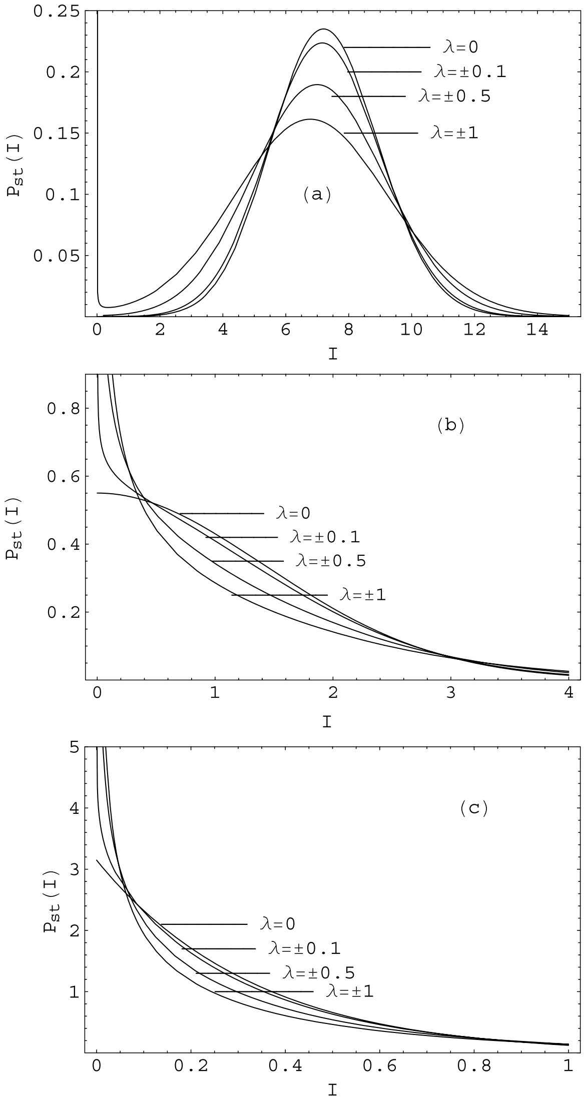


Fig. 2 $P_{st}(I)$ vs. the variable I for three different values of the pump parameter a_0. Parameters chosen are $A = 1$, $Q = 2$ and $K = 30$. (**a**) $a_0 = 6$. (**b**) $a_0 = 0$. (**c**) $a_0 = -6$

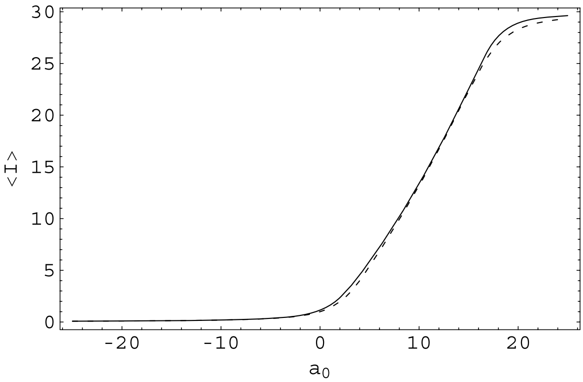


Fig. 3 The mean stationary laser intensity $\langle I \rangle$ as a function of pump parameter a_0 with $K = 30$, $A = 1$, $Q = 2$, $|\lambda| = 0.1$ (*solid line*), and $|\lambda| = 0.9$ (*dotted line*)

Fig. 4 The mean stationary laser intensity $\langle I \rangle$ as a function of the cross-correlation coefficient strength λ with $K = 30$, $A = 1$, and $Q = 2$. (**a**) $a_0 = 5$, $a_0 = 0$. (**b**) $a_0 = 0$, $a_0 = -5$

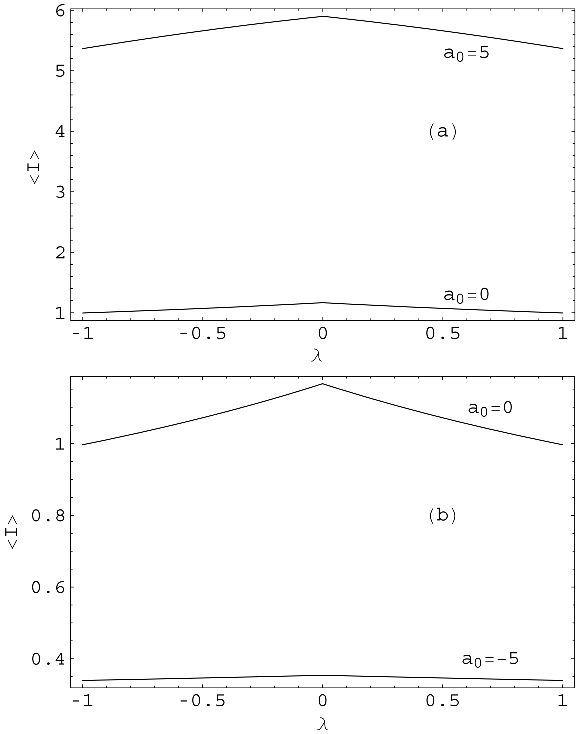
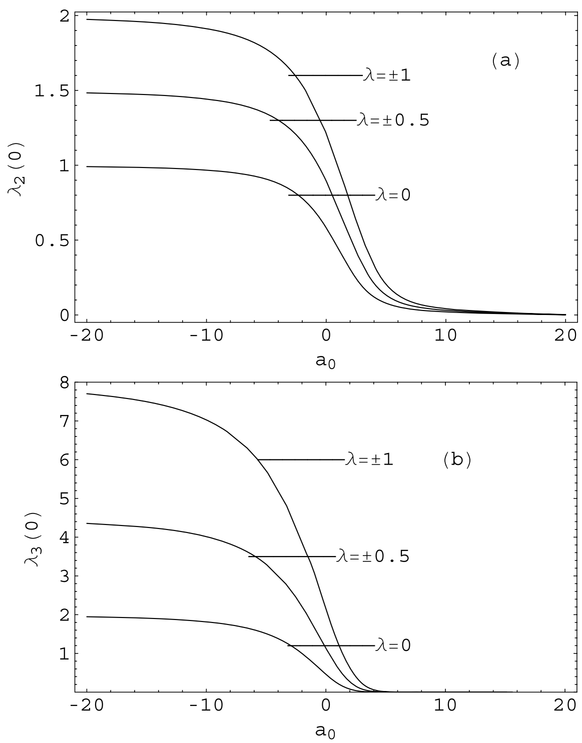


Fig. 5 The normalized variance $\lambda_2(0)$ and the normalized skewness $\lambda_3(0)$ of the steady-state laser intensity as functions of the pump parameter a_0. Parameters chosen are $A = 1$, and $Q = 2$. (**a**) The normalized variance $\lambda_2(0)$ of the laser intensity for different values of λ. (**b**) The normalized skewness $\lambda_3(0)$ of the laser intensity for different values of λ



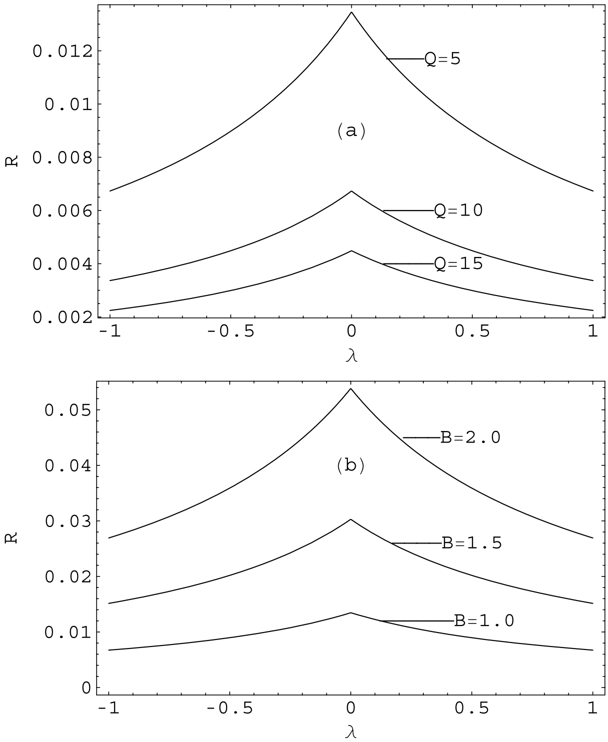


Fig. 6 The signal-to-noise ratio R versus the cross-correlation strength λ with $K = 30$, $A = 1$, and $a_0 = 5$. (**a**) $B = 1$, and Q take different values. (**b**) $Q = 5$, and B take different values

SNR as a function of the cross-correlation coefficient λ between $q_r(t)$ and $q_R(t)$ is plotted in Fig. 6. From Fig. 6 we see that the SNR distribution exhibits one-maximum structure and the SNR distribution possesses the maximum at the cross-correlation coefficient $\lambda = 0$. In Fig. 6(a), the maximum value of the peak of R decreases as the noise intensity Q increases. In Fig. 6(b), the maximum value of the peak of R increases as the amplitude of the periodic signal B increases.

From the above discussion, we understand that the cross-correlation between the real and imaginary parts of the quantum noise plays an important role in a laser model with a full account of the saturation effects. The cross-correlation strength $|\lambda|$ can make the first-order-like-transition occur. When the laser system is operated above the threshold, the mean $\langle I \rangle$ becomes larger, the output of the laser intensity increases, and the coupling strength $|\lambda|$ attenuates the output of the laser intensity. When the laser is operated near and below the threshold, the mean $\langle I \rangle$ becomes smaller, the output of the laser intensity decreases, and the coupling strength $|\lambda|$ still attenuates the output of the laser intensity. When the laser is operated at the threshold and below the threshold, the cross-correlation strength $|\lambda|$ enhances the fluctuation of the laser intensity, in which the fluctuation of the laser intensity mainly centers on positive fluctuation, the laser system can not generate the steady laser intensity output. When the laser is operated above the threshold, the fluctuations of the laser intensity are extremely suppressed whatever $|\lambda|$ changes, and the laser system can generate the steady laser intensity output. When a periodic signal is added to a saturation laser model with cross-correlation between quantum noise terms, the interesting stochastic resonance phenomena occur at $\lambda = 0$. The noise intensity Q decreases the values of the resonance peak, however, the amplitude of the periodic signal B enhances the values of the resonance peak.

Acknowledgement The author wishes to express his most sincere thanks to Referee and Editor, who read the manuscript carefully and give valuable advice, comments and help.

References

1. Haken, H., Roy, R.: Laser Theory. Springer, Berlin (1984)
2. Sargent III, M., Scully, M.O., Lamb Jr., W.E.: Laser Physics. Addison-Wesley, Reading (1974)
3. Loulsell, W.H.: Quantum Statistical Properties of Radiation. Willey, New York (1973)
4. Zhu, S., Yin, J.: Saturation effect in a laser at steady state. Phys. Rev. A **45**(7), 4969–4973 (1992)
5. Risken, H.: The Fokker-Planck Equation. Springer, Berlin (1984)
6. Fulnski, A., Telejko, T.: On the effect of interference of additive and multiplicative noises. Phys. Lett. A **152**(1–2), 11–14 (1991)
7. Zhou, X.P., Gao, W.J., Zhu, S.Q.: Fluctuations in a nonlinear field with coupling of additive noise terms. Phys. Lett. A **214**(3–4), 131–138 (1996)
8. Ke, S.Z., Cao, L., Wu, D.J., Yao, K.L.: Stationary properties in a single-mode laser with cross-correlation between quantum noise terms. Phys. Lett. A **281**(2–3), 113–118 (1996)
9. Jia, Y., Cao, L., Wu, D.J.: Effects of a dye laser with correlations between additive and multiplicative noise: Transient properties. Phys. Rev. A **51**(4), 3196–3202 (1995)
10. Madureira, A.J.R., Jung, P., Hanggi, P.: Dye laser with pump and quantum noise. Phys. Rev. A **54**(1), 755–759 (1996)
11. Quan, L., Cao, L., Wu, D.J., Li, Z.G.: One-dimensional model of a single-mode laser with correlation between additive and multiplicative noises. Phys. Lett. A **216**(1–5), 106–110 (1996)
12. Marchi, M., Marchesoni, M., Gammaitoni, L., Menichella-Saetta, E., Santucci, S.: Resonant activation in a bistable system. Phys. Rev. E **54**(4), 3479–3487 (1996)
13. Cao, L., Wu, D.J.: Cross-correlation of multiplicative and additive noises in a single-mode laser white-gain-noise model and correlated noises induced transitions. Phys. Lett. A **260**(1–2), 126–131 (1999)
14. Zhu, P.: Effects of self-correlation time and cross-correlation time of additive and multiplicative colored noises for dynamical properties of a bistable system. J. Stat. Phys. **124**(6), 1511–1525 (2006)
15. Xie, C.W., Mei, D.C., Zhou, L.Y.: Dynamical properties of a single-mode laser with correlations between real and imaginary parts of quantum noise. J. Optoelectron. Laser **15**(5), 621–624 (2004)
16. Zhang, L., Gao, L., Wu, D.J.: The research of single-mode laser driven by multiplicative colored noise with cross-correlation between the real and imaginary parts of the quantum noise. Physica A **332**, 207–218 (2004)
17. Zhu, S.: Saturation effects in a laser with multiplicative white noise. Phys. Rev. A **45**(5), 3210–3215 (1992)
18. Cao, L., Wu, D.J., Luo, X.L.: Effects of saturation in the transient process of a dye lase. I. White-noise case. Phys. Rev. A **45**(9), 6838–6847 (1992)
19. Cao, L., Wu, D.J., Luo, X.L.: Effects of saturation in the transient process of a dye laser. II. Colored-noise case. Phys. Rev. A **45**(9), 6848–6856 (1992)
20. Cao, L., Wu, D.J., Luo, X.L.: Effects of saturation in the transient process of a dye laser. III. The case of colored noise with large and small correlation time. Phys. Rev. A **47**(1), 57–70 (1993)
21. Zhu, P., Chen, S.B., Mei, D.C.: Intensity correlation function and associated relaxation time of a saturation laser model with correlated noises. Chin. Phys. Lett. **23**(1), 29–30 (2006)
22. Zhu, P., Chen, S.B., Mei, D.C.: Effects of correlated noises inf a saturation laser model. Mod. Phys. Lett. B **20**(23), 1481–1488 (2006)
23. Schaefer, R.B., Willis, C.: Quantum-mechanical theory of the organic-dye laser. Phys. Rev. A **13**(5), 1874-1890 (1976)
24. Baczynski, A., Kossakowski, A., Marzalek, T.: Quantum theory of dye lasers. Z. Phys. B **23**(2), 205–212 (1976)
25. Roy, R.: Photon distributions of lasers with first-order phase-transition analogies. Phys. Rev. A **20**(5), 2093–2104 (1979)
26. Roy, R., Mandel, L.: Dye laser statistics and the phase transition. Opt. Commun. **23**, 306–310 (1976)
27. Abate, J.A., Kimble, H.J., Mandel, L.: Photon statistics of a dye laser. Phys. Rev. A **14**(2), 788–795 (1976)
28. Lett, P., Gage, E.C., Chyba, T.H.: Colored-noise-induced first-order phase transition in a single-mode dye laser. Phys. Rev. A **35**(2), 746–751 (1987)
29. Aguado, M., San Miguel, M.: Analysis of a dye-laser mode including quantum noise. Phys. Rev. A **37**(2), 450–455 (1988)

30. Roy, R., Yu, A.W., Zhu, S.: In: Moss, F., McClintock, P.V.E. (eds.) Noise in Nonlinear Dynamical Systems. Cambridge University Press, Cambridge (1989)
31. Long, Q., Cao, L., Wu, D.J., Li, Z.G.: Phase lock and stationary fluctuations induced by correlation between additive and multiplicative noise terms in a single-mode laser. Phys. Lett. A **231**, 339–342 (1997)
32. Cao, L., Wu, D.J., Zhi, K.S.: Bistable kinetic model driven by correlation noises: Steady-state analysis. Phys. Rev. E **50**(4), 2496–2503 (1994)
33. Hernandez-Garcia, E., Toral, R., San Miguel, M.: Intensity correlation function for the colored gain-noise model of dye lasers. Phys. Rev. A **42**(11), 6823–6830 (1990)
34. Hernandez-Machado, A., San Miguel, M., Katz, S.L.: Intensity correlation function of dye lasers: Short-time behavior. Phys. Rev. A **31**(4), 2362–2365 (1985)
35. Zhang, L.Y., Cao, L., Wu, D.J., Wang, J.: Stochastic resonance in liner regime of a single-mode laser. Chin. Phys. Lett. **20**(1), 25–27 (2003)